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Economic Order Quantity Model With Exponential Demand Rate With Carbon Emission

[1] Varun Mohan , [2] Uttam Kumar Sharma , [3] Pervaiz Iqbal* , [4] Praveen Kumar

[1]Associate Professor, Department of Mathematics, Sharda University, Greater Noida, (UP), India.

[2]Assistant Professor, Department of Mathematics, MKR Government Degree College Nandgram Ghaziabad, India.

[3*]Assistant Professor, Department of Mathematics and Actuarial Science, B.S. Abdur Rahman Crescent Institute of Science & Technology, Chennai, India.

[4]Student, Department of Mathematics, Sharda University, Greater Noida, (UP), India.

Abstract

The biggest problem to deal with in 21st century is global warming. To control the impact to climatic changes governments have adopted climate friendly process in manufacturing sectors. Therefore it is important to consider the impact of carbon emission in EOQ (Economic Order Quantity) models. In our inventory system we have considered the carbon emission due to holding and deterioration of inventory. With all these presumptions a mathematical model is derived to find the optimal cycle time at which total inventory cost is minimum. To study the effect of change in parameters on total cost, sensitivity analysis is carried out with the help of a numerical example.

Keywords: EOQ Model, Exponential Rate of Demand, Carbon Emission, Deterioration.

1. Introduction

The inventory model's main objective is to minimize the cost. Demand is a very important factor in determining the correct level of inventory to order as demand never remains constant it varies with time depending on different factors like inflation, price of the product, preferences of

*Corresponding Author
pervaizmaths@gmail.com (Pervaiz Iqbal)

consumers, price of related products, and many other factors. Gupta et al. (2013) presented a model for items with variable demand rate, Weibull deterioration, and inflation (Singh, Gupta, & Gupta, 2013). Ray and Chaudhuri (1997) developed a model with stock-dependent demand, shortage, inflation, and time discounting (Ray & Chaudhuri, 1997). Sana (2011) discussed a model for decayable items with price-sensitive demand (Sankar Sana, 2011). Kumar et al. (2012) has established a model for limited storage and time-dependent demand under inflation (Kumar, Singh, & Kumari, 2012). Tripathi and Tomar (2015) discussed the results of temporary price discounts ordered by a supplier on retailer's replenishment policy (Tripathi & Tomar, 2015).

In the real world, situation deterioration could be a method that ends up in worsening of products with time. Different kinds of products possess different deterioration patterns. Several researchers have created a reference to deterioration in their inventory models. Wee, (1995) developed a joint pricing and replenishment policy for deteriorating products with the declining market (Wee, 1995). Dave (1986) planned a model for deteriorating products with lead time (Dave, 1986). Recently Mohan et al. many researchers like Karthikeyan and Santhi (2015) (Karthikeyan & Santhi, 2015) and Chang et al. (2003) (Chang, Ouyang, & Teng, 2003) With various other factors, climatic change is one of the major concerns on which firms are focusing. As the temperature is increasing globally, every sector is concerned about their carbon emission as carbon dioxide has a remarkable effect on global warming. In 2014 United States Environmental Protection Agency (EPA) indicated that the Industrial sector is the reason for almost 21% of the greenhouse gases emission in the United States. EPA also highlighted that new technologies and climate-sensitive processes can play a vital role in carbon emission reduction. Many international conferences were organized to address the issue of carbon emission and other greenhouse gases emission which are the main reason for the global increase in temperature. Imposition of carbon tax/carbon credit and Cap and Trade mechanism were the major steps towards reducing emission (EU, 2005) and (Ki-moon, 2008).

Therefore, while formulating the inventory models carbon emission is one of the factors which should not be ignored. Including carbon emission in the inventory management model is vital for any organization because of growing pressure from numerous International organizations, government, and consumer awareness concerning environmental degradation. To minimize carbon emissions, corporations have started adopting energy efficient equipment and facilities. Hua et al. (2011) compared the impacts of carbon cap and carbon price on order size, carbon emission, and total cost (Hua, Cheng, & Wang, 2011). Bouchery et al. (2012) have integrated sustainable development criteria in their multi objective formulation of EOQ (Bouchery, Ghaffari, Jemai, & Dallery, 2012). Recently As'ad et al. (2020) developed a lot-sizing model for a temperature-sensitive product while accounting for environmental constraints (As'ad, Hariga, & Shamayleh, 2020). Datta (2017) studied the effect of investment in green technology on a production inventory system (Datta, 2017).

Transportation is one of the major sources of carbon emission as more than 90% of the fuel used for transportation is consists of gasoline and diesel. Therefore, transportation is also an important factor to be considered while formulating a sustainable EOQ model. As'ad et al. (2020) have considered carbon emission due to transportation in their sustainable dynamic lot-sizing model for cold items. Arikan and Jammernegg, (2014) have studied a complex inventory model, where they have considered emission from transportation, manufacturing, and warehousing and compared different sourcing strategies for the possibility of the emission reduction (Arikan & Jammernegg, 2014).

In this paper, we have studied the EOQ model with constant deterioration and exponential rate of demand under the influence of Carbon emission. Rest of the paper is arranged as follows, Section 2 describes basic notations and assumptions which are used in this paper. The mathematical model of the proposed system is derived in section 3 where we derived the expressions for deteriorated items, initial ordering quantity, Carbon emission cost, and in section 4 we optimized the total cost. Section 5 contains a numerical example, Sensitivity analysis on different parameters is presented in section 6. Finally, a Discussion on results is presented in section 7 conclusion and directions for future research are contained in section 8.

2. Notations and Assumptions

2.1 Notations

Following notations are being used

A	:	Ordering cost
c	:	Unit purchase cost
h	:	Inventory holding cost per unit time
θ	:	Rate of deterioration, $0 \leq \theta < 1$
D(t)	:	$\left(ae^{bt}\right)$ Demand rate at time t , a is constant demand and b is rate of change of demand where $a > 0, b > 0$
q	:	maximum inventory level
T	:	Cycle time
HC	:	Holding cost
DC	:	Deterioration cost

CC : Cost of carbon emission

T* : Optimized cycle time

Z(T) : Total cost per cycle

Z*(T) : Optimized total cost per cycle

q* : Inventory level at optimal time T*

$\widehat{HC}$: Carbon emission due to holding of inventory

$\widehat{DC}$: Carbon emission due to deterioration of inventory

$\hat{h}$: Carbon emission due to holding inventory per unit time

$\hat{q}$: Carbon emission due to deterioration of inventory per unit time

δ : Amount of money paid as carbon tax for carbon emitted per unit time

2.2 Assumptions

1. Inventory system deals with single type of item only.
2. No shortages are allowed.
3. Zero lead time
4. Deterioration is constant
5. Demand is exponential

3. Mathematical Model

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t),\ 0 \le t \le T \tag{1}$$

With I(0) = q and I(T) = 0 The solution of (1) with condition I(T)

$$I(t) = -\frac{ae^{-t\theta}(e^{t(b+\theta)} - e^{T(b+\theta)})}{b+\theta} \tag{2}$$

Using I(0) = q

$$q = -\frac{a(1-e^{T(b+\theta)})}{b+\theta} \tag{3}$$

Holding cost during

$$\mathrm{HC} = h\int_0^T I(t)dt$$

$$= \frac{ah(e^{bT}(b(-1+e^{T\theta})-\theta)+\theta)}{b\theta(b+\theta)} \tag{4}$$

No. of units deteriorated

$$Q = \frac{a(e^{bT}-1)}{b}$$

Therefore,

Cost of deterioration

$$\mathrm{DC} = c(q - Q)$$

$$= c\left(-\frac{a(-1+e^{bT})}{b} - \frac{a(1-e^{T(b+\theta)})}{b+\theta}\right) \tag{5}$$

Carbon emission due to holding of inventory

$$\widehat{\mathrm{HC}} = \hat{h}\int_0^T I(t)dt$$

$$= \frac{a\hat{h}(e^{bT}(b(-1+e^{T\theta})-\theta)+\theta)}{b\theta(b+\theta)} \tag{6}$$

Carbon emission due to deterioration of inventory

$$\widehat{\mathrm{DC}} = \hat{q}(q - Q)$$

$$= \hat{q}\left(-\frac{a(-1+e^{bT})}{b} - \frac{a(1-e^{T(b+\theta)})}{b+\theta}\right) \tag{7}$$

Cost of carbon emission

$$\mathrm{CC} = \delta(\widehat{\mathrm{HC}} + \widehat{\mathrm{DC}})$$

$$= \delta\left(\frac{a\hat{h}(e^{bT}(b(-1+e^{T\theta})-\theta)+\theta)}{b\theta(b+\theta)} + \hat{q}\left(-\frac{a(-1+e^{bT})}{b} - \frac{a(1-e^{T(b+\theta)})}{b+\theta}\right)\right) \tag{8}$$

Total cost

The total cost Z per unit time of an inventory is

$$Z(T)=\frac{[A+HC+DC+CC]}{T}$$

$$=\left(\frac{1}{T}\right)\left[A+\frac{ah\left(e^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+\theta\right)}{b\theta(b+\theta)}+\ c\left(-\frac{a\left(-1+e^{bT}\right)}{b}-\frac{a\left(1-e^{T(b+\theta)}\right)}{b+\theta}\right)+\delta\left(\frac{a\hat{h}\left(e^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+\theta\right)}{b\theta(b+\theta)}+\hat{q}\left(-\frac{a\left(-1+e^{bT}\right)}{b}-\frac{a\left(1-e^{T(b+\theta)}\right)}{b+\theta}\right)\right)\right] \tag{9}$$

4. Optimized total cost

Equation (9) is used to find optimal solution, where total cost is minimum.

Differentiating (9) with respect to T

$$\frac{dZ(T)}{dT}=\frac{1}{T}\left(c\left(-ae^{bT}+ae^{T(b+\theta)}\right)+\frac{ah\left(be^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+be^{bT+T\theta}\theta\right)}{b\theta(b+\theta)}+\delta\left(\left(-ae^{bT}+ae^{T(b+\theta)}\right)\hat{q}+\frac{a\hat{h}\left(be^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+be^{bT+T\theta}\theta\right)}{b\theta(b+\theta)}\right)\right)$$
$$-\frac{1}{T^2}\left(A+\frac{ah\left(e^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+\theta\right)}{b\theta(b+\theta)}+c\left(-\frac{a\left(-1+e^{bT}\right)}{b}-\frac{a\left(1-e^{T(b+\theta)}\right)}{b+\theta}\right)+\delta\left(\frac{a\hat{h}\left(e^{bT}\left(b\left(-1+e^{T\theta}\right)-\theta\right)+\theta\right)}{b\theta(b+\theta)}+\hat{q}\left(-\frac{a\left(-1+e^{bT}\right)}{b}-\frac{a\left(1-e^{T(b+\theta)}\right)}{b+\theta}\right)\right)\right)$$

(10)

$$\frac{d^2Z(T)}{dT^2}=\frac{2Ab\theta(b+\theta)+a(h+\alpha\delta+(c+\beta\delta)\theta)(b^3e^{bT}(-1+e^{T\theta})T^2-2(-1+e^{bT})\theta+b^2e^{bT}T(2-T\theta+2e^{T\theta}(-\ \ +be^{bT}(-2+2T\theta+e^{T\theta}(2-2T\theta+T^2\theta^2)))}{bT^3\theta(b+\theta)}$$

(11)

$$\frac{d^2Z(T)}{dT^2} > 0$$

This shows that optimal solution gives the minimum value.

The optimal values of T = T* is obtained by solving $\frac{dZ(T)}{dT} = 0$, we get optimal (minimum) values of T = T*.

NOTE: Mathematica 9.0 software is used to compute optimal value T = T* and Z(T) = Z*(T)

5. Numerical Example

Let us consider an inventory system with following data:

A	:	300 units per year
a	:	150 units per year
b	:	1 units per year
c	:	$ 30 per year
θ	:	0.01
h	:	$10 per year
δ	:	$ 5 per year
$\hat{h}$	:	3 units per year
$\hat{q}$	:	5 units per year

We get T = T* = 0.321123 year and Z(T) = Z*(T) = $1699.47

6. Sensitivity Analysis

A sensitivity analysis is performed to study the effects of changes in parameters on the calculated optimal cost. We observe the changes in the optimal decision values, when only one parameter varies and other remains unchanged. Following tables shows the effect of chance in parameters on total cost, initial inventory level and time.

Table 1 Sensitivity analysis table

Parameter	value	T = T*	Z = Z*
A	310	0.325524	1730.4
	320	0.329833	1760.91
	330	0.334056	1791.04
	340	0.338195	1820.79
	350	0.342255	1850.18
a	160	0.312606	1749.65
	170	0.304781	1798.26
	180	0.297554	1845.45
	190	0.290854	1891.33
	200	0.284615	1936
b	2	0.277785	1858.12
	3	0.247991	2002.44
	4	0.225743	2136.69
	5	0.208255	2263.35
	6	0.194016	2383.97
c	35	0.320862	1700.96
	40	0.320602	1702.46
	45	0.320343	1703.95
	50	0.320084	1705.45
	55	0.319826	1706.94
h	12	0.311197	1758.23
	14	0.302199	1814.86
	16	0.293992	1869.58
	18	0.286462	1922.57
	20	0.279519	1973.98
θ	0.02	0.318026	1716.69
	0.03	0.315017	1733.75
	0.04	0.312092	1750.64
	0.05	0.309247	1767.36
	0.06	0.306478	1783.93
δ	6	0.306367	1788.21
	7	0.2936	1872.27
	8	0.282403	1952.32
	9	0.272473	2028.88
	10	0.263581	2102.37
	4	0.298005	1842.44

$\hat{h}$	5	0.279519	1973.98
	6	0.264275	2096.45
	7	0.251407	2211.51
	8	0.240346	2320.34
$\hat{q}$	6	0.320862	1700.96
	7	0.320602	1702.46
	8	0.320343	1703.95
	9	0.320084	1705.45
	10	0.319826	1706.94

7. Results and Discussion

- When Ordering cost (A) increases, the Optimal total cost (Z*) and Optimal cycle time (T*) increases. That is the change in A will lead to positive change in Z* and T*.
- When 'a' and 'b' increases, the Optimal cycle time (T*) decreases and Optimal total cost (Z*) increases. That is the change in a will lead to negative change in T* while positive change in Z*.
- When 'c' and 'h' increases T* decreases while Z* increases. That is change in 'c' and 'h' will lead to positive change in Z* and negative change in T*.
- When (θ) and (δ) increases T* decreases while Z* increases. That is change in θ and δ will lead to positive change in Z* and negative change in T*.
- When ($\hat{h}$) and ($\hat{q}$) increases T* decreases while Z* increases. That is change in ($\hat{h}$) and ($\hat{q}$) will lead to negative change in T* while positive change in Z.

8. Conclusion

Here we have an inventory model for deteriorating items with exponential demand. We have examined the cost caused due to carbon emission in holding and deterioration of inventory along with holding and deterioration cost. A mathematical model is derived of the inventory system to find optimal cycle time at which total cost is minimum. Then we have discussed how changes in parameter are affecting optimal total cost and optimal cycle time. Parameters like rate of deterioration (θ), carbon emission cost (δ), carbon emission due to holding ($\hat{h}$), ordering cost (A), initial demand (a), 'b' has significant effect on the total cost per cycle time.

This model can further be extended in the light different carbon taxation schemes like cap and trade mechanism, time dependent deterioration can also be taken with this model or one may consider different type of demand functions.

Mathematica 9.0 software is used to find optimal solution T = T*, Z(T) = Z*(T).

References

Arikan, E., & Jammernegg, W. (2014). The single period inventory model under dual sourcing and product carbon footprint constraint. International Journal of Production Economics, 157(1), 15–23. https://doi.org/10.1016/j.ijpe.2014.04.029

As'ad, R., Hariga, M., & Shamayleh, A. (2020). Sustainable dynamic lot sizing models for cold products under carbon cap policy. Computers and Industrial Engineering, 149(July). https://doi.org/10.1016/j.cie.2020.106800

Bouchery, Y., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Including sustainability criteria into inventory models. European Journal of Operational Research, 222(2), 229–240. https://doi.org/10.1016/j.ejor.2012.05.004

Chang, C. T., Ouyang, L. Y., & Teng, J. T. (2003). An EOQ model for deteriorating items under supplier credits linked to ordering quantity. Applied Mathematical Modelling, 27(12), 983–996. https://doi.org/10.1016/S0307-904X(03)00131-8

Datta, T. K. (2017). Effect of Green Technology Investment on a Production-Inventory System with Carbon Tax. Advances in Operations Research, 2017. https://doi.org/10.1155/2017/4834839

Dave, U. (1986). A probabilistic scheduling period inventory model for deteriorating items with lead time. Zeitschrift Für Operations Research, 30(5), 229–237. https://doi.org/10.1007/BF01919292

EU. (2005). EU action against climate change: EU emission trading - an open scheme promoting global innovation. 1 TS-RIS M4-Citavi.

Hua, G., Cheng, T. C. E., & Wang, S. (2011). Managing carbon footprints in inventory management. International Journal of Production Economics, 132(2), 178–185. https://doi.org/10.1016/j.ijpe.2011.03.024

Karthikeyan, K., & Santhi, G. (2015). An inventory model for constant deteriorating items with cubic demand and salvage value. International Journal of Applied Engineering Research, 10(55), 3723–3728.

Ki-moon, B. (2008). Kyoto Protocol Reference Manual. United Nations Framework Convention on Climate Change, 130. https://doi.org/10.5213/jkcs.1998.2.2.62

Kumar, N., Singh, S. R., & Kumari, R. (2012). An inventory model with time-dependent demand and limited storage facility under inflation. Advances in Operations Research, 2012.

https://doi.org/10.1155/2012/321471

Ray, J., & Chaudhuri, K. S. (1997). An EOQ model with stock-dependent demand, shortage, inflation and time discounting. International Journal of Production Economics, 53(2), 171–180. https://doi.org/10.1016/S0925-5273(97)00112-6

Sankar Sana, S. (2011). Price-sensitive demand for perishable items - An EOQ model. Applied Mathematics and Computation, 217(13), 6248–6259. https://doi.org/10.1016/j.amc.2010.12.113

Singh, S. R., Gupta, V., & Gupta, P. (2013). 463 EOQ Model with Volume Agility, Variable Demand Rate, Weibull Deterioration Rate and Inflation. International Journal of Computer Applications, 72(23), 1–6.

Tripathi, R. P., & Tomar, S. S. (2015). Optimal order policy for deteriorating items with time-dependent demand in response to temporary price discount linked to order quantity. International Journal of Mathematical Analysis, 9(21–24), 1095–1109. https://doi.org/10.12988/ijma.2015.5235

Wee, H. M. (1995). Joint pricing and replenishment policy for deteriorating inventory with declining market. International Journal of Production Economics, 40(2–3), 163–171. https://doi.org/10.1016/0925-5273(95)00053-3